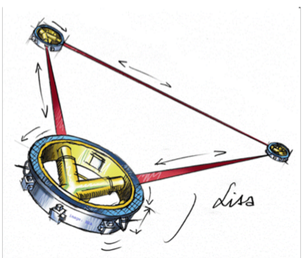

The Impact of Merger & Ringdown on non-spinning SMBHB Parameter Estimation with LISA

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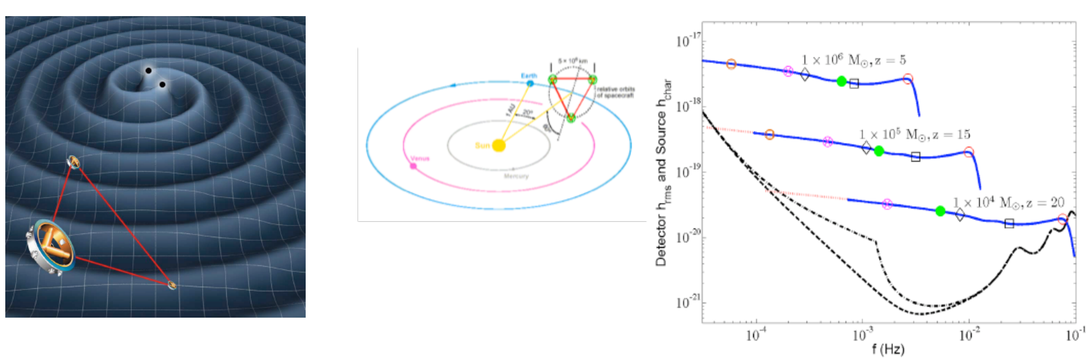
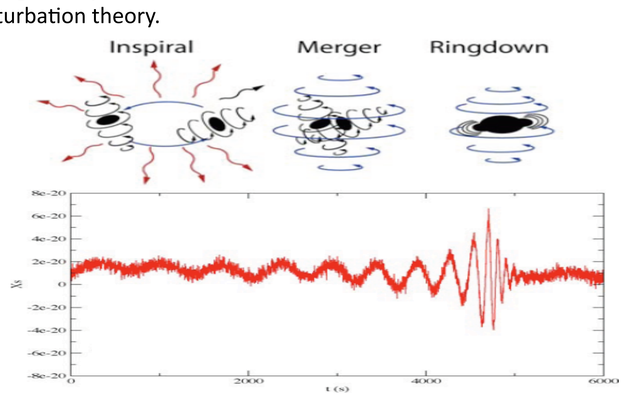
Abstract: The Laser Interferometer Space Antenna (LISA) will measure gravitational waves from the inspiral and merger of supermassive black hole binaries (SMBHBs) at high redshift with large signal to noise. These measurements will allow extraction of the SMBHB parameters (component masses, spins, binary orbital parameters, sky location, and distance) with exquisite accuracy. Here we present a study of the impact on measured parameter precision from the inclusion of accurate waveforms for the merger/ringdown portion of the signal. We focus specifically on sky-position and luminosity distance, the most important parameters for constraining searches for potential electromagnetic counterparts to SMBHB merger events.

LISA Science & Instrument: LISA is a joint effort of NASA and ESA to build a space-based detector of gravitational waves in the milliHertz band, a regime that is expected to be rich in astrophysical sources including close compact binaries in the Milkyway, capture of compact objects by massive black holes in the nearby Universe, and mergers of supermassive black holes at high redshifts. LISA will use laser interferometry to measure the *~10^{-12}m* tidal distortions induced by these waves on a triangular constellation of satellites *$5x10^9$ m* on a side.



Left: LISA will detect gravitational waves by measuring distortions in a triangular detector formed by three spacecraft linked with laser beams. Center: The LISA constellation trails the Earth in its heliocentric orbit, with the plane of the detector inclined at 60o to the ecliptic. Right: LISA's sensitivity to equal mass, non-spinning SMBHB events (blue lines) is limited by instrumental noise (dashed line) and foreground sources (dot-dashed line).

Black Hole Binary Waveforms: The gravitational-wave driven coalescence of a pair of black holes produces a waveform known as a *chirp,* characterized by increasing frequency and amplitude. For convenience, the waveform is often divided into three epochs differing in the approaches used to model them. The initial epoch, *inspiral*, consists of the adiabatic decay of quasi-Keplerian orbits, a process well-described by the post-Newtonian (PN) formalism. The middle epoch, from the inner-most stable circular orbit (ISCO) to the coalescence of the event horizons is known as the *merger* and has historically only been accessible via numerical relativity. The final epoch, in which the distorted event horizon settles into a quiescent Kerr state through damped oscillations of the BH's normal modes is termed the *ringdown* and can be treated with BH perturbation theory.



Top: Cartoon depiction of three epochs of black hole binary coalescence.
Bottom: Simulated LISA observation of a merger of two non-spinning $10^6\ M_\odot$ black holes at z = 15. Note that the full instrumental and foreground noises are simulated in this plot, the SNR is approximately 200.

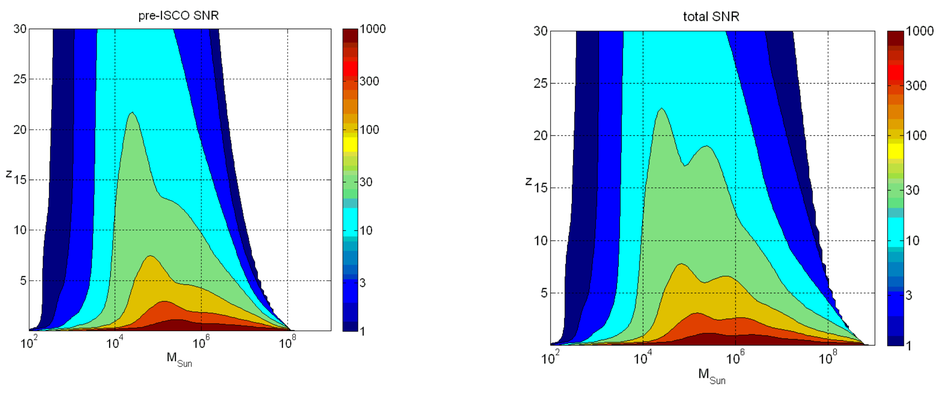
Parameter Estimation: For typical systems, LISA will be able to measure the last year of inspiral, the merger, and the ringdown. The full signal can have a SNR in excess of 1000, allowing LISA to make precise measurements of the astrophysical parameters that control the shape of the waveforms. As astrophysical systems, black hole binaries are remarkably "clean" in that they depend on a limited number of parameters. The most general system is completely described by 17 parameters: Two black hole masses, six quantities describing the spin vectors of the two black holes, two parameters describing the eccentricity of the binary, three angles describing its orientation, two angles describing its position on the sky, the time of coalescence, and the absolute (luminosity) distance. It is expected that SMBHBs will be circularized prior to entering the LISA band, which reduces the number of parameters encoded in the observed waveforms to 15.

One common way to estimate LISA's ability to estimate astrophysical parameters of SMBHBs ("parameter estimation estimation") is to use the Fisher matrix approach. For a system in which a measurement X depends on a set of parameters Λ^i, the Fisher matrix Γ^{ij} is defined as:

$$\Gamma^{ij} \equiv \left(\frac{\partial X}{\partial \Lambda^i} \middle| \frac{\partial X}{\partial \Lambda^j} \right) \qquad (X|Y) \equiv 4\mathcal{R} \int_0^\infty \frac{\tilde{X}^\star(f) \cdot \tilde{Y} f)}{S(f)} df$$

Where tilde denotes Fourier transform and *S(f)* is the noise spectral density of the measurement. For large SNR, the Fisher matrix can be used to directly compute the covariance matrix and hence the parameter variances

$$\Sigma^{ij} = \left(\Gamma^{ij}\right)^{-1} \left[1 + \mathcal{O}\left(SNR^{-1}\right)\right] \qquad \sigma^i \equiv \sqrt{\Sigma^{ii}}$$



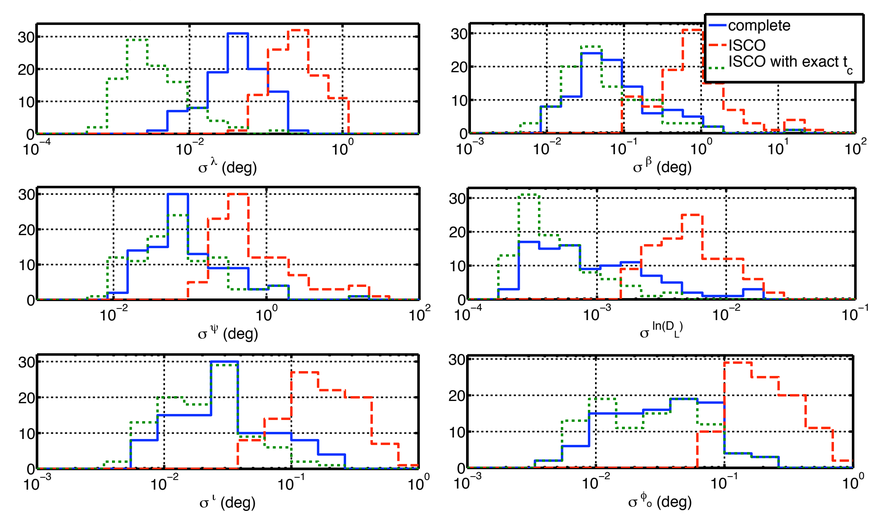
Comparison of LISA SNR for equal-mass, non-spinning SMBHB observations versus redshift and total mass for the inspiral epoch only (left) and total signal (right).

Methodology: For this study we used a semi-analytic waveform family termed the implicit rotating source / effective-one-body (IRS-EOB) [PRD **78** 044046 (2008)] to model the complete inspiral, merger, and ringdown of a non-spinning SMBHB. Our canonical system is an equal-mass, non-spinning SMBHB with a total rest mass of $2x10^6\ M_\odot$ at a redshift of *z = 1,* for which we compute the final year of the waveform. The software package *Synthetic LISA* [PRD **71** 022001 (2005)] is used to compute the LISA observables from the strain in the solar system barycenter (SSB). The three LISA Michelson-type time-delay interferometry *(TDI)* variables (*X,Y,* and *Z*) are used to form three orthogonal observables (*A, E,* and *T*).

The Fisher matrix is computed using two-sided finite differencing, with each component in the difference being a repeat of the above process with a slight change to one of the waveform parameters. The covariance matrix and parameter variances are then computed for this particular system using the approach outlined in the previous section. To explore differences in sensitivity for systems with different nominal parameters, we Monte Carlo our original system parameters over sky position, orientation, and merger time.

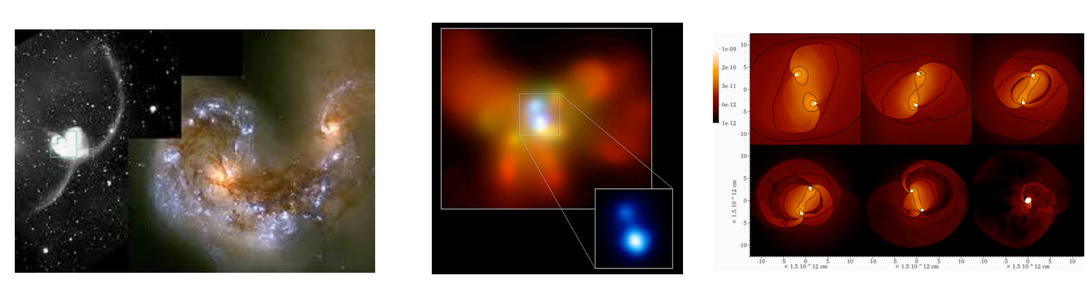
Results: In general we find that, for the systems studied, LISA will be able to measure the parameters with high precision. For example, the sky location of the source will be localized to 0.1° or better for a typical system. When compared with similar estimates that neglect the contribution from the merger, we find that the improvement in parameter accuracy with the inclusion of the merger is roughly proportional to the increase in SNR. This suggests that the "information density" of the merger signal is roughly equivalent to that of the inspiral.

One possible explanation for this improvement is that the merger can be used to place tight constraints on the time of coalescence by virtue of the fact that it contains a sharp peak that is absent in the inspiral. Since the coalescence time is correlated with the other parameters to various degrees, an improvement in its measurement can improve the precision of all parameters. This picture is supported by the fact that the measured precision of the full signal is comparable to that of the inspiral if the error in the coalescence time is artificially set to zero.



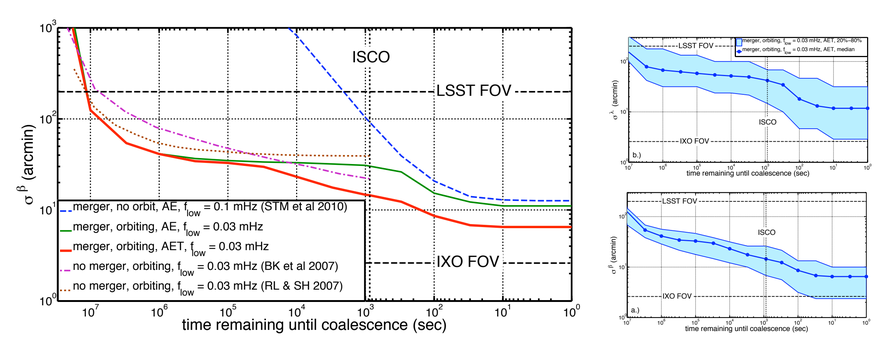
Parameter estimation precision for total waveform, inspiral waveform, and inspiral waveform with exact knowledge of coalescence time. Left-to-right, top-to-bottom the parameters are ecliptic longitude, ecliptic latitude, polarization phase, (log) luminosity distance, inclination angle, and orbital phase. Note that the discrepancy between the blue and green curves for ecliptic latitude is an artifact caused by our definition of coalescence time (SSB vs. detector)

Electromagnetic Counterparts: One of the most interesting applications of LISA's ability to localize SMBHB sources is that it could enable coordinated observations of a SMBHB merger event with both gravitational waves and electromagnetic waves. Determining the existence and nature of electromagnetic counterparts is an active area of research, but there are several plausible mechanisms that could produce observable signals that are roughly contemporaneous with the peak GW emission.



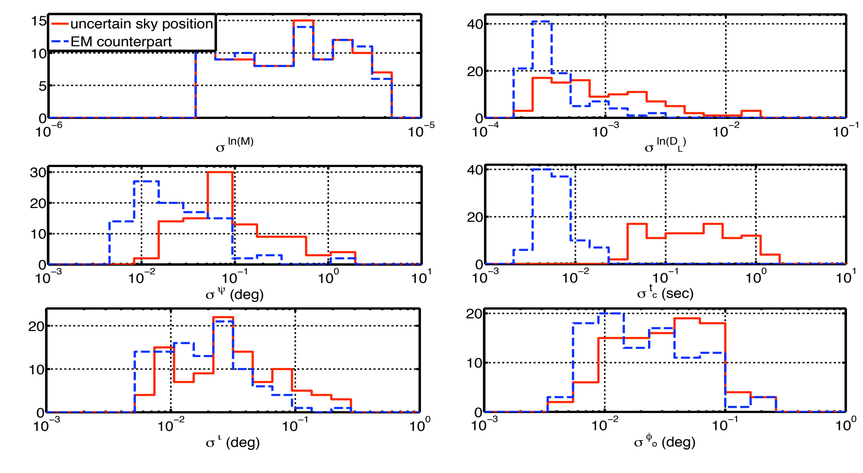
Left: Hubble image of antennae galaxies, a merging system. Center: Chandra image of pair of black holes in a recently merged galaxy. Right: Simulations of accretion around a merging binary black hole (from Bode, et al. ApJ **715**,2, 1117 (2010))

LISA will detect SMBHBs of the type we studied roughly one year prior to coalescence. The initial observations will provide very little sky localization information. However, as the observations continue, the localization information will improve. The rate at which this information accumulates will be important for planning coordinated observations and allocating appropriate resources.



Improvement in sky localization (β = ecliptic latitude, λ = ecliptic longitude) as a function of time remaining to coalescence. The left plot shows the median results for calculations including various effects while the right show the distribution of our ensemble. The field of view of two representative EM instruments (LSST & IXO) are shown for comparison.

An EM counterpart will provide very precise sky localization information. When added to the GW observations with LISA, additional improvements in the other parameters can be realized.



Improvement in parameter estimation with "exact" sky localization, assumed to be provided by an EM counterpart. Left-to-right, top-to-bottom the parameters are (log) total mass, (log) luminosity distance, polarization phase, coalescence time (at SSB), orbital inclination, and orbital phase.